Name____ Period____

Work Energy Packet 4

Work-Kinetic Energy Theorem / Conservation of Mechanical Energy Review

$$W = F * \Delta d \qquad PE = mgh \qquad KE = \frac{1}{2}mv^2 \qquad W = \Delta E \qquad E_i = E_f$$

In this universe, energy is always conserved, meaning total energy does not change as time marches on ($E_i = E_f$). But it is very difficult to completely account for all forms of energy (kinetic, thermal, radiant, sound, electrical potential, mass, etc). So in this course, we only account for two types: kinetic energy ($\frac{1}{2}mv^2$) and gravitational potential energy (mgh). Everything else, we lump into a "Work" term. Then we say Work equals a CHANGE of energy or W = ΔKE. These problems ultimately come down the following question:

Do I use the Work Kinetic Energy Theorem (W=ΔKE) or Conservation of Mechanical Energy ($E_i = E_f$)?

Here's how to answer it

Energy Problem Solving Strategy:

1: Draw the combined FBD, initial final drawing, also showing the direction of motion (displacement) **d**
2: Do trig as needed to make all forces on your FBD either:
  a. Parallel to **d**, or
  b. Perpendicular to **d**
3: Circle the forces or components of any forces **parallel** to the direction of motion **d**
4: Choose the **appropriate Energy Equation** as follows

If there are no forces circled, or only F_g, is circled, then you may use Conservation of Mechanical Energy ($E_i = E_f$):

$$E_i = E_f$$
$$PE_i + KE_i = PE_f + KE_f$$
$$\cancel{m}a_g h_i + \tfrac{1}{2}\cancel{m}(v_i)^2 = \cancel{m}a_g h_f + \tfrac{1}{2}\cancel{m}(v_f)^2$$

or

If you have any non-F_g force(s) circled, you must use Work Kinetic Energy Theorem (W=ΔKE):

$$W = \Delta KE$$
$$\Sigma F \cdot d = \Delta KE.$$
$$\Sigma F \cdot d = KE_f - KE_i$$
$$\Sigma F \cdot d = \tfrac{1}{2}m(v_f)^2 - \tfrac{1}{2}m(v_i)^2$$

Where ΣF is the TOTAL force in the direction of **d**

In this course, energy problems come in several types: *(Examples using this strategy for some of them follow)*
level, ramp, roller coasters, free fall and pendulum

Level
1. With or without friction
2. With or without pull (which may be angled)

Ramp
1. Without friction
2. With or without pull (which is limited to parallel to surface)

Roller Coaster
Without friction

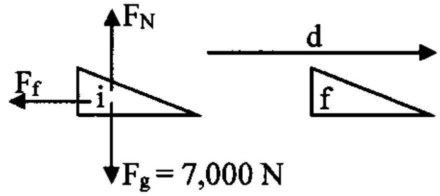
Freefall:
No air resistance

Pendulum:
No friction

In any of these problems, you may be asked to find:
a force, speed, distance, coefficient of friction, height, or mass

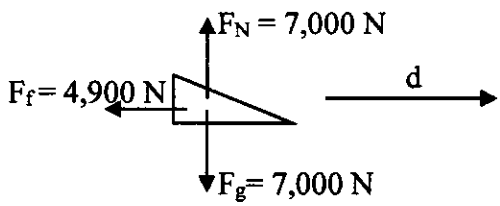
Example 1: A 700 kg car going 20 m/s skids to a stop on a level road. The coefficient of friction is 0.7. What distance does it take to stop?

Step 1: Draw the combined FBD, initial final drawing including **d**:



Step 2: Do trig as needed to make all forces on your FBD either:

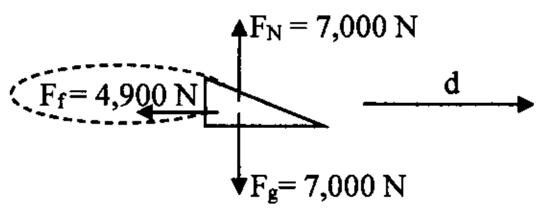
a. Parallel to the direction of motion, or
b. Perpendicular to the direction of motion



No trig needed. Forces already are perpendicular or parallel to d.

From: $F_{Ty} = 0$
$F_N + F_g = 0$
$F_N - 7{,}000\text{ N} = 0$
$F_N = 7{,}000\text{ N}$
And $F_f = \mu F_N = 0.7(7{,}000) = 4{,}900\text{ N}$

Step 3: Circle the components of any forces **parallel** to the direction of motion d



Step 4: Choose the appropriate Energy Equation and solve for the desired variable.

Since you have a non-F_g force circled (F_f), you must use Work Kinetic Energy theorem ($W = \Delta KE$)

$$W_T = \Delta KE$$

$$F_T d = KE_f - KE_i$$

$$F_{Tx}\Delta x = \tfrac{1}{2}m(v_f)^2 - \tfrac{1}{2}m(v_i)^2$$

$F_{Tx} = F_f$

$$F_f\Delta x = \tfrac{1}{2}m(v_f)^2 - \tfrac{1}{2}m(v_i)^2$$

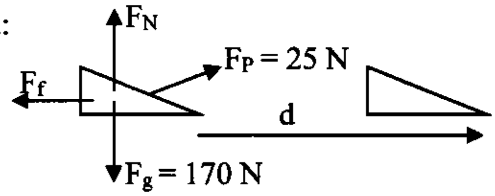
Negative because F_f points left

$$-4{,}900\Delta x = \tfrac{1}{2}(700)(0)^2 - \tfrac{1}{2}(700)(20)^2$$

$$\Delta x = 28.57\text{ m}$$

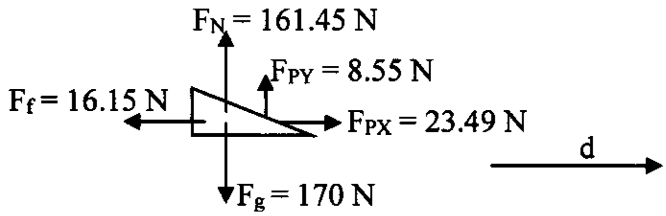
Example 2: You pull a 17 kg sled across the level snow from rest with a rope angled 20° above horizontal. You pull with a force of 25 N for a distance of 6 m across the snow and the coefficient of friction between the sled and the snow is 0.1. How fast is the sled going at the end of the 6 m?

Step 1: Draw the combined FBD, initial final drawing including **d**:



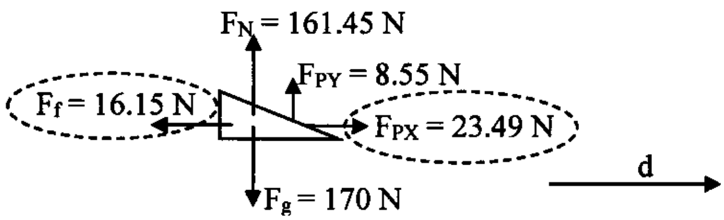
Step 2: Do trig as needed to make all forces on your FBD either:

a. Parallel to the direction of motion, or
b. Perpendicular to the direction of motion



$F_{PX} = F_P\cos(\theta) = 25\cos(20^\circ) = 23.49$ N
$F_{PY} = F_P\sin(\theta) = 25\sin(20^\circ) = 8.55$ N
From $F_{Ty} = 0$, $F_N + F_{PY} + F_g = 0$
$F_N + 8.55 - 170 = 0$
$F_N = 161.45$ N
And $F_f = \mu F_N = 0.1(161.45) = 16.15$ N

Step 3: Circle the components of any forces **parallel** to d



Step 4: Decide the appropriate Energy Equation and solve for the desired variable.

Since at least 1 non-F_g is force circled, you must use work Kinetic Energy Theorem (W = ΔE)

F_f is negative because it's pointing left, and F_{PX} is positive since it's pointing right. So:

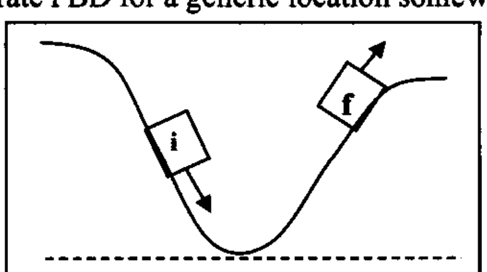
$F_{Td} = F_{TX}$ (since $d = \Delta x$)
$F_{Tx} = F_{PX} + F_f$ (from FBD)
$F_{Tx} = 23.49 - 16.15$
$F_{Tx} = 7.34$ N

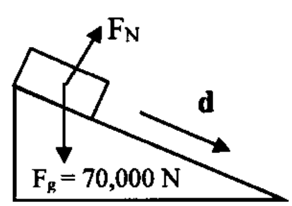
$W = \Delta KE$
$F_{Td} \cdot d = KE_f - KE_i$
$F_{Tx}\,\Delta x = \frac{1}{2}m(v_f)^2 - \frac{1}{2}m(v_i)^2$
$7.34(6) = \frac{1}{2}(17)(v_f)^2 - \frac{1}{2}(17)(0)^2$
$v_f = 2.28$ m/s

$v_i = 0$ since it starts from rest.

Example 4: A 7,000 kg frictionless roller coaster is traveling at 18 m/s when it is part way down a hill such that it is 15 m above the ground. How fast is it going when it has climbed the next hill to a height of 21 m above the ground?

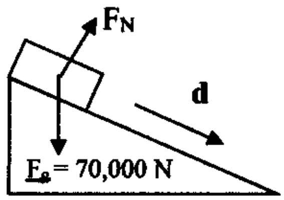
Step 1: Draw the FBD, initial final drawing, and **d**
(You could draw the FBD directly on the initial, final drawing; or as I have done below, draw a separate FBD for a generic location somewhere along the path of motion)





Step 3: Circle the components of any non- Fg forces **parallel** to **d**

There aren't any!

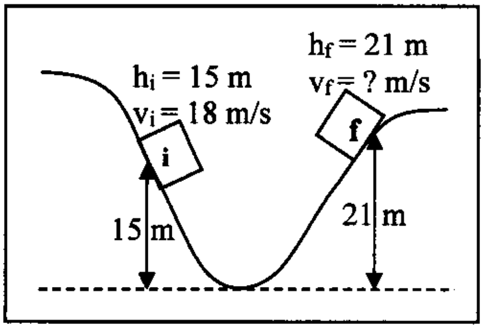


You don't know the angle of the slope, so you can't actually do the trig, meaning you can't put any numbers on any forces.

Nonetheless, perform this step anyway so you can think about the direction of the forces compared to the direction of **d**.

Step 4: Choose the appropriate Energy Equation and solve for the desired variable.

Since there aren't any circled forces, you may use conservation of energy, $E_i = E_f$.



$$\overbrace{PE_i + KE_i}^{E_i} = \overbrace{PE_f + KE_f}^{E_f}$$

$$\cancel{m}a_g h_i + \tfrac{1}{2}\cancel{m}(v_i)^2 = \cancel{m}a_g h_f + \tfrac{1}{2}\cancel{m}(v_f)^2$$

$$(10)(15) + \tfrac{1}{2}(18)^2 = (10)(21) + \tfrac{1}{2}(v_f)^2$$

$$v_f = 14.28 \text{ m/s}$$

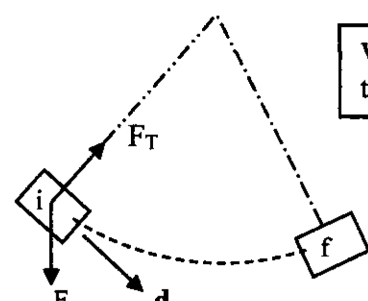
Since each term has an "m", you can divide through by m, which removes each m.

Example 5: You pull a pendulum back such that the "bob" at the end of the string is 0.075 m above the tabletop. You release it from that position and it arcs through the bottom and goes up the other side. How fast is it going when it is 0.060 m above the tabletop on the other side of its arc?

Step 1: Draw the FBD, initial final drawing, and **d**

Step 2: Only Fg goes partially in the path's direction

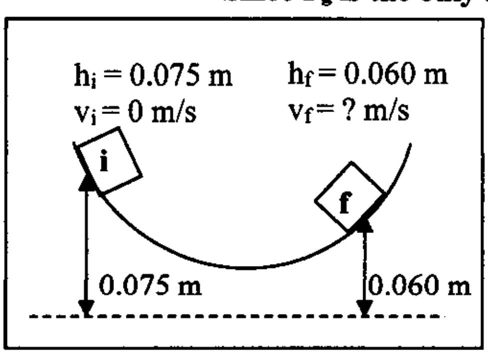
Step 3: The only non-Fg force is perpendicular to the path!



Where F_T is the force of the string on the "bob"

Step 4: Choose the appropriate Energy Equation and solve for the desired variable.

Since F_g is the only force doing work, you may use conservation of energy, $E_i = E_f$.



$$\overbrace{PE_i + KE_i}^{E_i} = \overbrace{PE_f + KE_f}^{E_f}$$

$$\cancel{m}a_g h_i + \tfrac{1}{2}\cancel{m}(v_i)^2 = \cancel{m}a_g h_f + \tfrac{1}{2}\cancel{m}(v_f)^2$$

$$(10)(0.075) + \tfrac{1}{2}(0)^2 = (10)(0.060) + \tfrac{1}{2}(v_f)^2$$

$$v_f = 0.55 \text{ m/s}$$

Since each term has an "m", you can divide through by m, which removes each m.

At the point at which you release a pendulum, its velocity is 0 m/s. (also true at the highest point either side of the arc)

Energy 4 Practice Questions

Work Kinetic Energy Theorem / Conservation of Mechanical Energy Review

(answers at end)

Do not use kinematics.

1. A 0.16 kg hockey puck slides to a stop along the level ice in 20 m. During that time, nothing is touching the puck except the ice. The coefficient of friction between the puck and the ice is 0.15. How fast was it going at the beginning of the 20m?

2. Nanook pulls his 12 kg sled across the level ice at a leisurely pace of 1 m/s with a horizontal force. Sensing an approaching storm, he begins to run. He pulls harder (still horizontally) such that the sled speeds up from 1 m/s to 2 m/s in a straight line distance of 3 m. The coefficient of friction between the sled and the ice is 0.2. How strong is the pulling force as the sled is speeding up?

3. With a force of 50 N, you pull an 8 kg box across the level floor in a straight line using a strap angled 20° above horizontal. The box speeds up from 1.2 m/s to 1.8 m/s in a distance of 3.4 m. What is the coefficient of friction between the floor and the box?

4. You throw a ball straight down from the roof of a building with an initial speed of 3 m/s. It is going 16 m/s just before it hits the ground. How tall is the building?

5. A 7.13 kg block of ice slides down a smooth metal ramp which is angled at 7° above horizontal. There is no friction.
   a. If the block is released from rest at the top of the ramp, how fast is it going when it has traveled 10 m along the ramp? (10 m is the hypotenuse of the triangle)

   b. Same scenario as the basic question (no friction), except this time, the block is given a gentle push such that it is already moving at 0.2 m/s at the top of the ramp. How far along the hypotenuse of the ramp must it travel to reach 1.5 m/s?

6. A 26,172.83 kg roller coaster is traveling 3 m/s at the top of the first hill which is 32 m above the ground. As it goes over the top of the next hill, it is traveling 12 m/s. Ignore friction.

How high is the second hill?

7. A pendulum is going 1.7 m/s at the bottom of its arc. How high above the bottom of its arc does the pendulum reach?

Answers:

1.	7.75 m/s
2.	30 N
3.	0.71
4.	12.35 m
5.a.	4.94 m/s
5.b.	0.90 m (+/- .01 ish due to trig rounding?)
6.	25.25 m
8.	0.14 m